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RECENT PROGRESS IN THE GROSS THEORY OF β-DECAY

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Abstract: The gross theory of β-decay has recently been improved. First, the effect of partial occupation of single-particle states in the pairing theory has been taken into account. Second, the function to represent the effect of the Pauli exclusion principle has been modified in order to observe the sum rule for the Fermi transition more accurately. Third, the single-particle strength function for the Gamow-Teller transition has been modified from consideration of the experimental data on Gamow-Teller giant resonances in (p,n) reactions; about 40% of the strength has been moved from the giant resonance region to the far tail part. Some single-particle strength functions for the first forbidden transitions have also been modified correspondingly. Results of calculations for β strength functions, β-decay half-lives, and average β-ray and γ-ray energies are discussed, and it is shown that the above modification really improve the agreement between theory and experiment on the average. Also discussed are methods for predicting deviations of the properties of individual nuclides from the average. In particular, a simple method is proposed for estimating the β-decay half-life of a nuclide from seven quantities, i.e., experimental half-lives of three neighboring nuclides and calculated half-lives of these four nuclides.

(β-decay, half-life, β strength function, average β-ray energy, average γ-ray energy, nuclear systematics)

I. Introduction

The gross theory of β-decay was constructed more than fifteen years ago by Takahashi, Yamada, and their collaborators[1~4] to describe gross properties of nuclear β-decay. The basic formula of this theory is an expression to give the β-strength function as an integral as

$$|M_\Omega(E)|^2=\int_{\varepsilon_{min}}^{\varepsilon_{max}} D_\Omega(E,\varepsilon)W(E,\varepsilon)\frac{dn_1}{d\varepsilon}d\varepsilon\,. \tag{1}$$

Here, Ω denotes the type of β-transition (Fermi, Gamow-Teller, etc.), E represents the energy of the final nuclear state measured from the parent nucleus, ε is the single-neutron (proton) energy for β^--decay (β^+-decay and electron capture), $dn_1/d\varepsilon$ is the neutron (proton) energy distribution in the parent, and $D_\Omega(E,\varepsilon)$ is a single-particle strength function. This single-particle strength function is defined as the function representing the hypothetical probability of energy change E which would be caused by the operation of the single-particle β-decay operator of type Ω if there were no Pauli exclusion principle. Finally, $W(E,\varepsilon)$ is a weight function to take into account the Pauli principle.

Now the problem is how to determine the three functions appearing in the integrand of Eq.(1). In the gross theory of Takahashi, Yamada and Kondoh,[4] it was done as follows. The two functions $W(E,\varepsilon)$ and $dn_1/d\varepsilon$ were determined on the basis of a simplified pairing model,[2] in which the pairing gaps are taken into account but the effect of the pairing factors (usually denoted by U and V) is not. The single-particle strength functions $D_\Omega(E,\varepsilon)$ were obtained as slight modifications of smooth functions $D^{(0)}_\Omega(E)$, which are schematically illustrated in Fig.1, and this modification was to take into account the pairing gaps.[2] By assuming appropriate functional forms for $D^{(0)}_\Omega(E)$, the whole theory was constructed including one free parameter, which represents the width of $D^{(0)}_\Omega(E)$ for the Gamow-Teller transition, $D^{(0)}_{GT}(E)$. The value of this free parameter was determined from comparison of calculated and experimental half-lives, and the resulting theoretical half-lives were given in extensive figures in 1973.[4] This particular version of the gross theory has long been used as a standard gross theory; we hereafter refer to it as the TYK version of the gross theory.

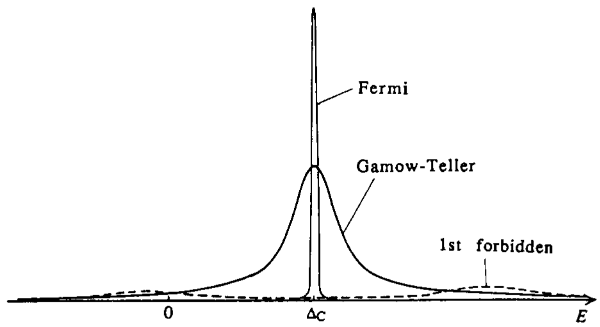

Fig. 1 Schematic illustration of the smoothed single-particle strength functions $D^{(0)}_\Omega(E)$ for the Fermi, Gamow-Teller, and first forbidden transitions in the TYK version.[4] This figure is for β^--decay, with the Coulomb displacement energy Δ_C being positive. For β^+-decay, Δ_C is negative.

After that time, we have had many new knowledges about the nuclear β-decay and the β strength function. The most remarkable one is the experimental confirmation of the existence of the concentration of the Gamow-Teller strength in the vicinity of the isobaric analog state (IAS), which was observed as a giant resonance in the (p,n) reaction.[5] However, the concentration of the strength was not

100%; it was only about 50% of the sum-rule value. Then, where is the other half?

Another information has come from an accumulation of less spectacular data; it has gradually become clear that the newly measured β-decay half-lives, in particular those of far neutron-rich nuclei, are systematically shorter than the gross-theory values. In this respect, the microscopic calculation by Klapdor et al.[6] published in 1984 seems to give a better tendency.

A few other undesirable points were also found in the TYK version. Therefore, we have undertaken to improve it. The improvement has been made in three points, of which the first two were discussed by Kondoh et al.,[7] and the final results including the third improvement were presented by Tachibana et al. at the 5th Conference on Nuclei Far From Stability.[8] We hereafter refer to this version as the improved version of the gross theory.

In the next section we give a brief sketch of the improvement, and in Section III we discuss the results of the improved version of the gross theory. Section IV is devoted to a few attempts to discuss or predict peculiarities of individual nuclides keeping close contact with the gross theory.

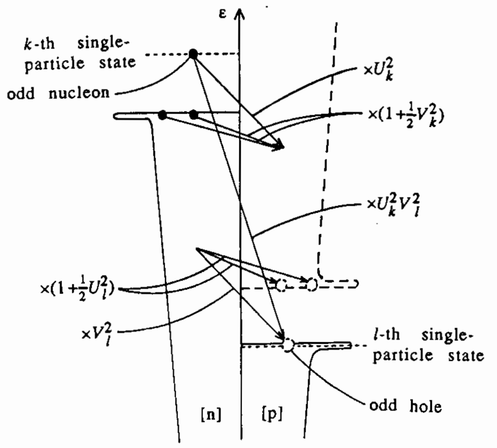

Fig. 2 Illustration of the shift of β strengths. The parts surrounded by solid lines are filled by paired nucleons except for the indicated odd hole. The part surrounded by dashed line is vacant. The strengths for the transitions from the odd nucleon or to the odd hole are reduced as shown, and they are shifted to the transitions from the highest paired nucleons or to the lowest "paired" holes.

II. Improvement of the Gross Theory

In improving the gross theory, we have kept Eq.(1) as the expression for the β strength function. As mentioned, the improvement has been made in three respects. The first is related to the effect of the pairing, the second to the sum rule for the Fermi transition, and the third to the single-particle strength function for the Gamow-Teller transition. In this section we only give a brief explanation for them; for more details, see Refs. 7 and 8.

2.1 Partial occupation of single-particle states

It is generally admitted that the ground state of an even-even nucleus has a structure similar to the BCS ground state. In the simplest picture of this structure, any two single-particle states having mutually opposite orbital and spin angular momenta are both occupied with a certain probability V_k^2 and both vacant with the probability U_k^2 $(=1-V_k^2)$, where k distinguishes the single-particle states. Let us refer to one of these two single-particle states as the partner state of the other. In the ground state of an odd-Z nucleus (Z: proton number), the odd proton always occupies a certain single-particle state, while its partner state is always vacant. A similar statement holds for an odd-N nucleus (N: neutron number). The β-decay causes a transition between an odd-Z nucleus and an even-Z nucleus and between an odd-N nucleus and an even-N nucleus. If this transition starts from the odd proton (or the odd neutron), the above-mentioned difference between the structures of even-Z and odd-Z (or even-N and odd-N) nuclei causes a retardation of the transition as discussed by many authors. A similar statement holds if the β-transition ends by filling the odd proton-hole (or the odd neutron-hole), where the odd hole means the partner state of the odd proton or odd neutron.

It was shown in Ref.7 that the effect of this partial occupation of single-particle states including the above-mentioned retardation can be taken into account in the gross theory by shifting a certain fraction of the strength associated with the odd nucleon or odd hole to transitions from nearby paired states or transitions to nearby paired holes as illustrated in Fig.2. This shift reduces the strength in the high transition-energy part. In the formulation of the gross theory, this improvement is performed as a change in the process of determining $D_\Omega(E,\varepsilon)$ from $D_\Omega^{(0)}(E)$ (or $D_\Omega^{(0)}(E,\varepsilon)$).

The value of U_k^2 and V_k^2 may be chosen suitably for each nuclide, but in the following we simply use their certain average value 1/2.

2.2 Observation of the sum rule for the Fermi transition

As was stressed in many places, sum rules played an essential role in building up the idea of the gross theory. However, it turned out that the strength function given by Eq.(1) is not always compatible with the sum rules. Actually, the TYK version violates the simplest sum rule for the Fermi transition by about 40%, the calculated values being too large. In Ref.7 a method was proposed for removing this discrepancy on the average by modifying the weight function $W(E,\varepsilon)$. Since $W(E,\varepsilon)$ is independent of the type of transition Ω, this modification reduces all kinds of strengths by about 30%.

2.3 Single-particle strength function

The Gamow-Teller strength in the TYK version is concentrated in the vicinity of IAS with a width of several MeV. Experimental studies of the Gamow-Teller giant resonances in (p,n) reactions[5] revealed that only about half of the Gamow-Teller strength is concentrated, and the peak position of the concentration is not exactly the position of IAS, the former being higher than the latter approximately by

$$\Delta_\tau = -30(N-Z)/A+6.7 \text{ MeV}. \qquad (2)$$

Taking into account these experimental data, we assume the smoothed single-particle strength function for the Gamow-Teller transition $D_{\mathrm{GT}}^{(0)}(E,\varepsilon)$ to be composed of two parts, one with a width of several MeV and the other with a much wider width. Specifically, we have taken the following function

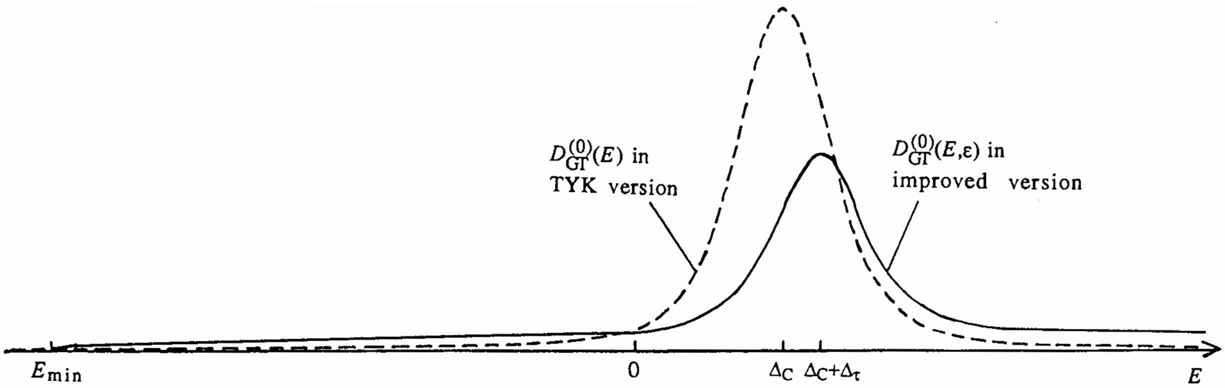

Fig. 3 Schematic illustration of the smoothed single-particle strength functions for the Gamow-Teller transition in the improved version of the gross theory as compared with that in the TYK version. This particular example applies to β^--decay of $N-Z$ top neutrons; for the lower energy neutrons, Δ_τ should be put equal to zero. For β^+-decay, Δ_C is negative, and $\Delta_\tau=-30(Z-N)/A+6.7$MeV for $Z-N$ top protons and $\Delta_\tau=0$ for the other protons.

as a first approximation of $D^{(0)}_{GT}(E,\varepsilon)$,

$$D^{00}_{GT}(E)=C_1\mathrm{sech}[\pi(E-\Delta)/2\sigma_1]+C_2\mathrm{sech}[\pi(E-\Delta)/2\sigma_2]\ , \quad (3)$$

where Δ represents the position of the peak, and is taken either as the Coulomb displacement energy Δ_C or as $\Delta_C+\Delta_\tau$ depending on the single-particle energy of the decaying nucleon (see caption of Fig.3). Taking into account the results of (p,n) reactions,[5] we take the smaller width parameter as $\sigma_1=[(4\mathrm{MeV})^2+\sigma_C^2]^{1/2}$, where σ_C is a small quantity given by Eq.(22) of Ref.4. The hyperbolic secant function has a character somewhere between the Gaussian and modified-Lorentz functions used in the TYK version.

In order to get a rough idea about the width of the wider distribution (σ_2) in Eq.(3), we have made calculations of sum rules for the Gamow-Teller single-particle strength function by the cluster variation method using two simple central nucleon-nucleon potentials.[9,10] The results are*

$$\frac{1}{3}\int_{E_{min}}^{\infty}(E-\Delta_C)D_{GT}(E,\varepsilon)dE=\begin{cases}1\ \ \mathrm{MeV}\ \ \mathrm{OMY}^{9}\\ 1\ \ \mathrm{MeV}\ \ \mathrm{YOY}^{10}\end{cases} \quad (4)$$

$$\frac{1}{3}\int_{E_{min}}^{\infty}(E-\Delta_C)^2D_{GT}(E,\varepsilon)dE=\begin{cases}(38\ \ \mathrm{MeV})^2\ \ \mathrm{OMY}\\ (207\ \ \mathrm{MeV})^2\ \ \mathrm{YOY}\end{cases} \quad (5)$$

Since the potentials used are not very realistic, these results have little quantitative value, but Eq.(5) certainly suggests that the Gamow-Teller strength is distributed very widely. Then, we take somewhat arbitrarily as $\sigma_2=[(135\mathrm{MeV})^2+\sigma_C^2]^{1/2}$.

The distribution $D^{00}_{GT}(E)$ was further modified slightly so as to inhibit the transition proceeding to the energy region below the lowest single-particle state,[8] and this process gives $D^{(0)}_{GT}(E,\varepsilon)$. The coefficients C_1 and C_2 in Eq.(3) are chosen so that they give the total normalization as $\int_{E_{min}}^{\infty}D^{(0)}_{GT}(E,\varepsilon)dE=3$, and the narrower distribution contributes 60% to this normalization. With these specifications, the sum-rule values corresponding to Eqs.(4) and (5) are approximately equal to 30 MeV and (80 MeV)^2, respectively. The former might seem to be too large compared to the value of Eq.(4), but it is likely that non-central forces greatly increase this value. The resulting function $D^{(0)}_{GT}(E,\varepsilon)$ is schematically illustrated in Fig.3.

Wide spreading of the single-particle strength function is also expected for forbidden transitions caused by axial-vector interaction. Therefore, for the first forbidden transitions caused by axial-vector interaction, we take an appropriate superposition of two forms of Eq.(3), i.e. a superposition of four hyperbolic secant functions.[8] The number of peaks is two as in the TYK version, but a widely distributed part is present in the improved version. Then, a procedure similar to that applied to the Gamow-Teller transition gives $D^{(0)}_{\Omega}(E,\varepsilon)$ for these first forbidden transitions.

No wide spreading of the single-particle strength function is expected for the transitions caused by vector interaction. Therefore, the only change we make for these transitions is the use of hyperbolic secant functions instead of Gaussian or modified-Lorentz functions.

III. Properties of the Improved Gross Theory

In this section, we discuss the properties of the improved version of the gross theory. The properties discussed are the β strength function, the β-decay half-life, and the average β-ray and g-ray energies. Delayed neutron emission will be discussed in a separate paper.

3.1 β strength function

In Figs.4~6 we show, for several selected nuclides, calculated allowed-equivalent total β strength functions defined by

$$S_\beta(E_{exc})=\rho(E_{exc})[\overline{1/f(Q-E_{exc})t}\,]\ , \quad (6)$$

where E_{exc} is the excitation energy of the daughter nucleus, $\rho(E_{exc})$ is the level density, $f(Q-E_{exc})$ is the

* These results are different from values given in Ref.8, because, after submitting the manuscript of Ref.8, we have found a mistake in the calculation. The values in Eqs.(4) and (5) are corrected ones. As the purpose of these calculations is simply to demonstrate qualitatively that the Gamow-Teller strength is widely distributed, we need not change the other part of Ref.8.

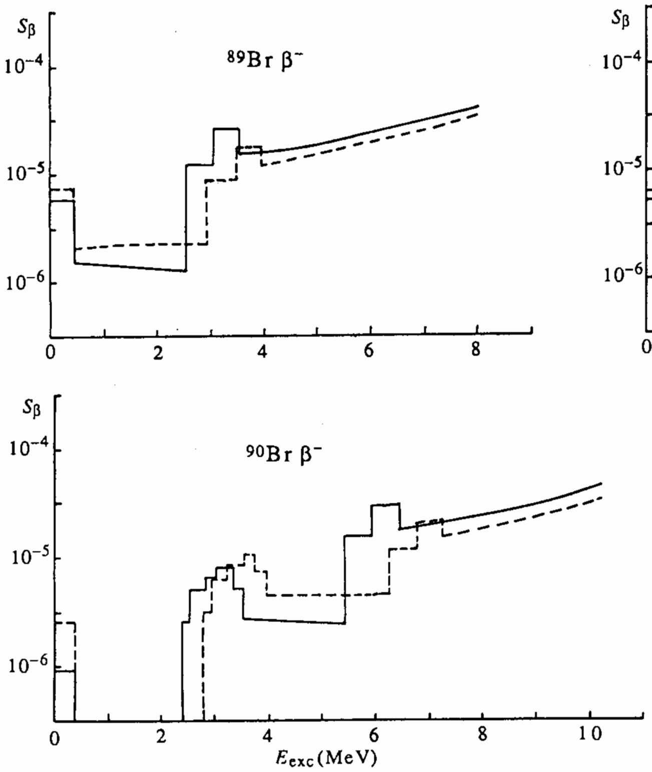

Fig. 4 Examples of calculated allowed-equivalent β^--decay strength functions in the region of light fission products. Solid line: improved version. Dashed line: TYK version. δ-functions appearing in the calculated strength functions are drawn with a width of 0.5 MeV for the ground state and 1 MeV for the other states.

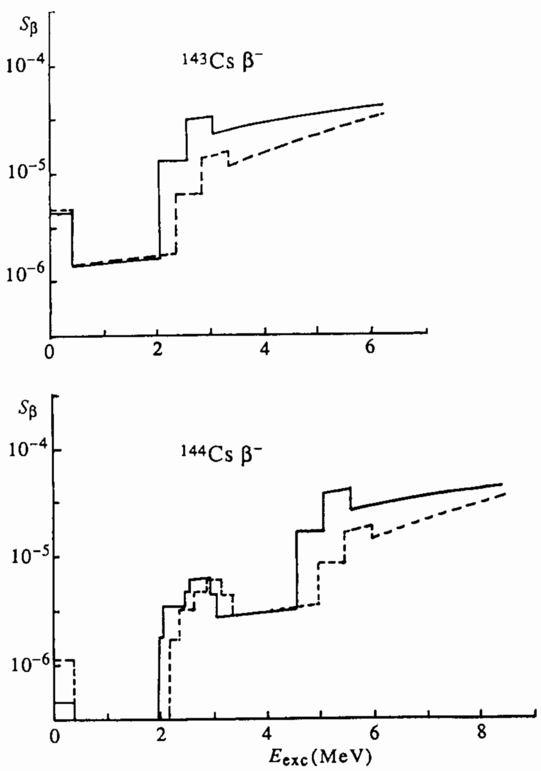

Fig. 5 Examples of calculated allowed-equivalent β^--decay strength functions in the region of heavy fission products. See also caption of Fig.4.

integrated Fermi function for allowed transitions, and $\overline{1/f(Q-E_{exc})t}$ is an appropriate average of the inverse ft values. The discontinuities in the low-excitation parts seen in Figs.4~6 are due to pairing gaps. Above those parts, the differences between the solid lines (improved version) and dashed lines (TYK

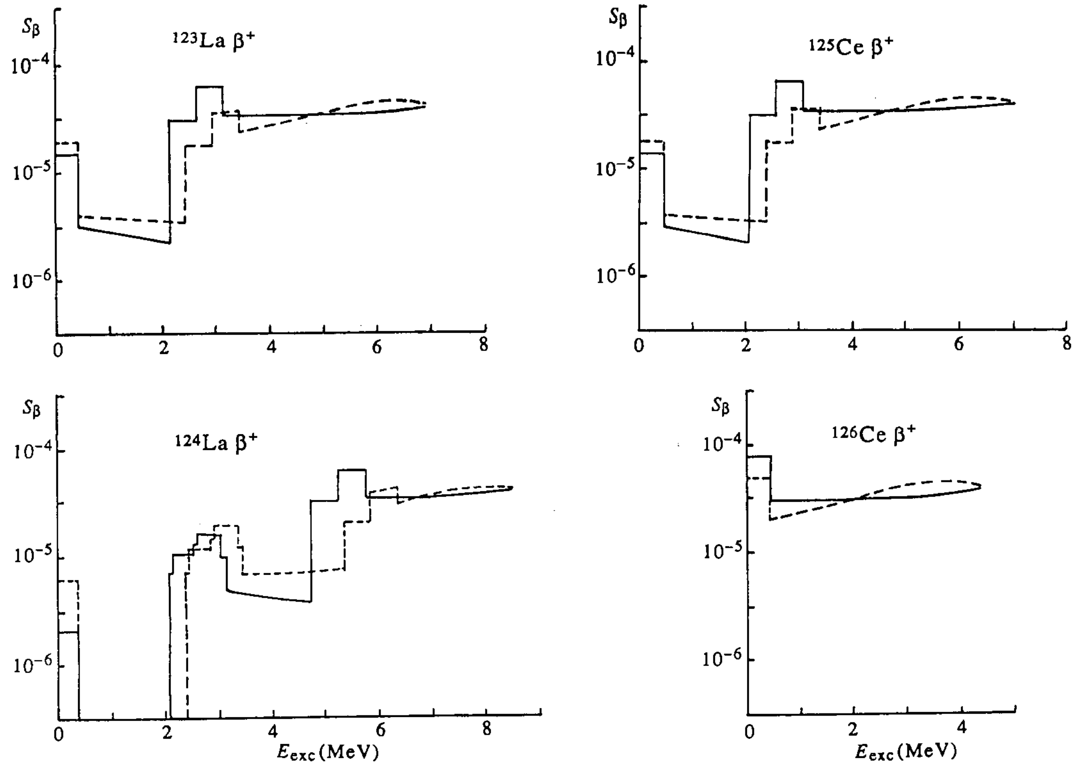

Fig. 6 Example of calculated allowed-equivalent β^+-decay strength functions in the region of heavy fission products. See also caption of Fig.4.

version) mainly reflect the differences between the corresponding single-particle strength functions. We also note that, in the lower-energy part of the gap region, the strength in the improved version is relatively small. This is due to the effect of the *UV* factors explained in Subsection 2.1.

3.2 β-decay half-life

The β-decay half-life is sensitive to the Q-value. We use the Q-values obtained from the mass values of Wapstra et al.[11] or, if they are not available, the Q-values calculated from the mass formula of Tachibana et al.[12]

The β-decay half-life is also sensitive to the forbiddenness of the transitions to low-lying states. In order to take into account this sensitivity, a procedure which may be called "bottom raising" was introduced.[1,4,8] This procedure is to shift all the strength in the region $0 \leq E_{exc} < \Delta Q$ to a single position $E_{exc}=\Delta Q$, which would be appropriate if the transition to the levels in the region $0 \leq E_{exc} < \Delta Q$ are highly forbidden and a strong transition occurs to a level at $E_{exc}=\Delta Q$. This problem will be discussed again in the next subsection.

In Fig.7 we show the general tendency of the ratios between β-decay half-lives calculated from the improved version and the TYK version of the gross theory. The magnitude of "bottom raising" is taken, in both versions, as

$$\Delta Q = \begin{cases} 0.25 \text{ MeV} & \text{for even-even parent} \\ 1 \text{ MeV} & \text{for odd-mass parent} \\ 1.75 \text{ MeV} & \text{for odd-odd parent.} \end{cases}$$

It is seen from this figure that the improved version gives shorter half-lives for far neutron-rich nuclei, a desirable tendency to fit to experimental data. This is due to the widely spread Gamow-Teller strength function. It is also noted that the β^+-decay half-lives of lighter nuclei are longer in the improved version.

In Fig.8 we compare the half-lives calculated in the improved version with experimental half-lives.[13] The same "bottom raising" as above is applied. It is seen from this figure that, as the nuclides move farther from the β stability line, the agreement between the calculated and experimental values becomes better; no large-scale discrepancy is found. The underestimation of half-lives for nuclei with $Z \geq 83$ and $N \leq 125$ is due to high forbiddenness of the transitions to many low-lying states, which is caused by the double magicity of ^{208}Pb.[14]

3.3 Average β-ray and γ-ray energies

Average β-ray and γ-ray energies ($\overline{E}_\beta$ and $\overline{E}_\gamma$) are important in the decay-heat problem. Yoshida and Nakasima[15] studied this problem with the help of the TYK version of the gross theory, and succeeded in correcting the overestimation of $\overline{E}_\beta$ and the corresponding underestimation of $\overline{E}_\gamma$ which were a source of trouble at that time. Then, isn't there any unwelcome possibility that the improved version destroy their success?

We have not yet done such an extensive study as did by Yoshida and Nakasima, but we are rather optimistic about this problem. In the improved version of the gross theory, the long tail of the Gamow-Teller strength function increases $\overline{E}_\beta$ and reduces $\overline{E}_\gamma$, while the effect of the *UV* factors reduces $\overline{E}_\beta$ and increases $\overline{E}_\gamma$. Our preliminary calculations suggest that the latter change predominates in most fission products. Therefore, if we follow exactly the procedure of Yoshida and Nakasima, we shall possibly get some underestimation of $\overline{E}_\beta$ and overestimation of $\overline{E}_\gamma$. However, their procedure is

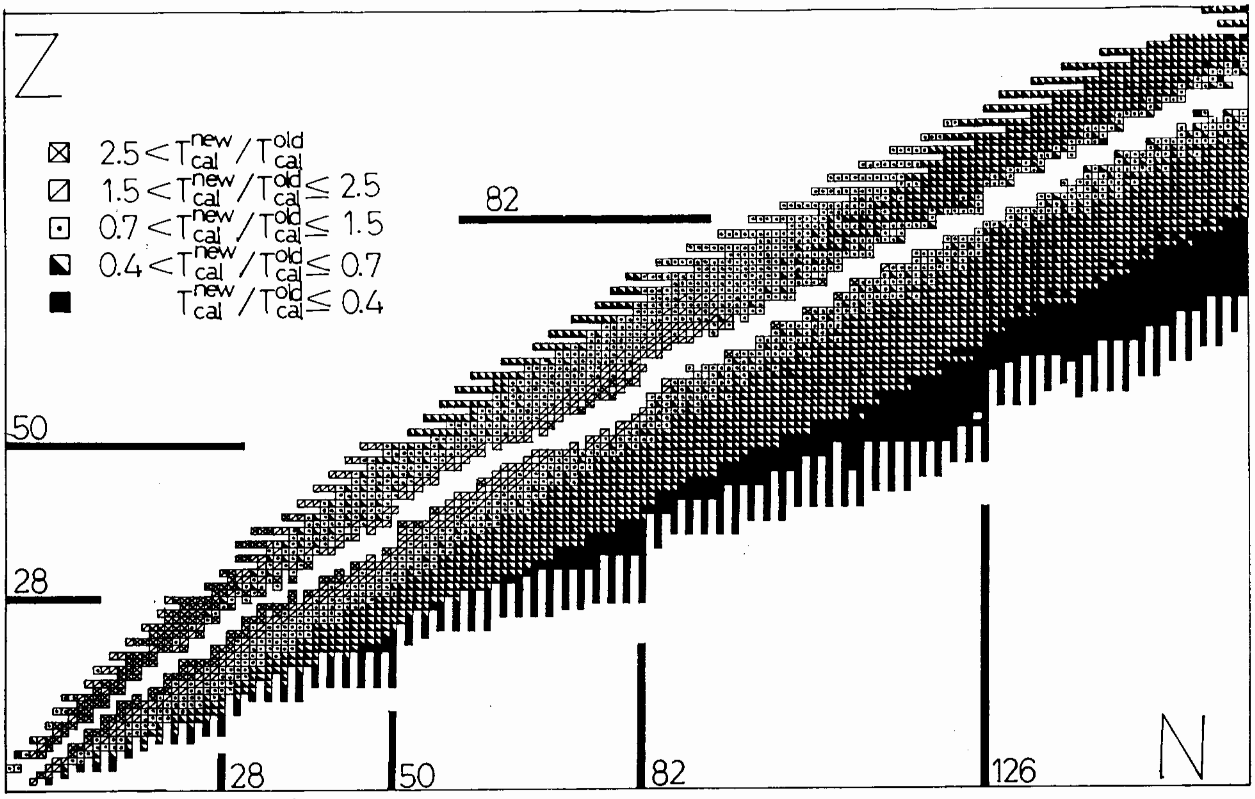

Fig. 7 General trend of the ratios between the β-decay half-lives calculated by the improved version and the TYK version of the gross theory.

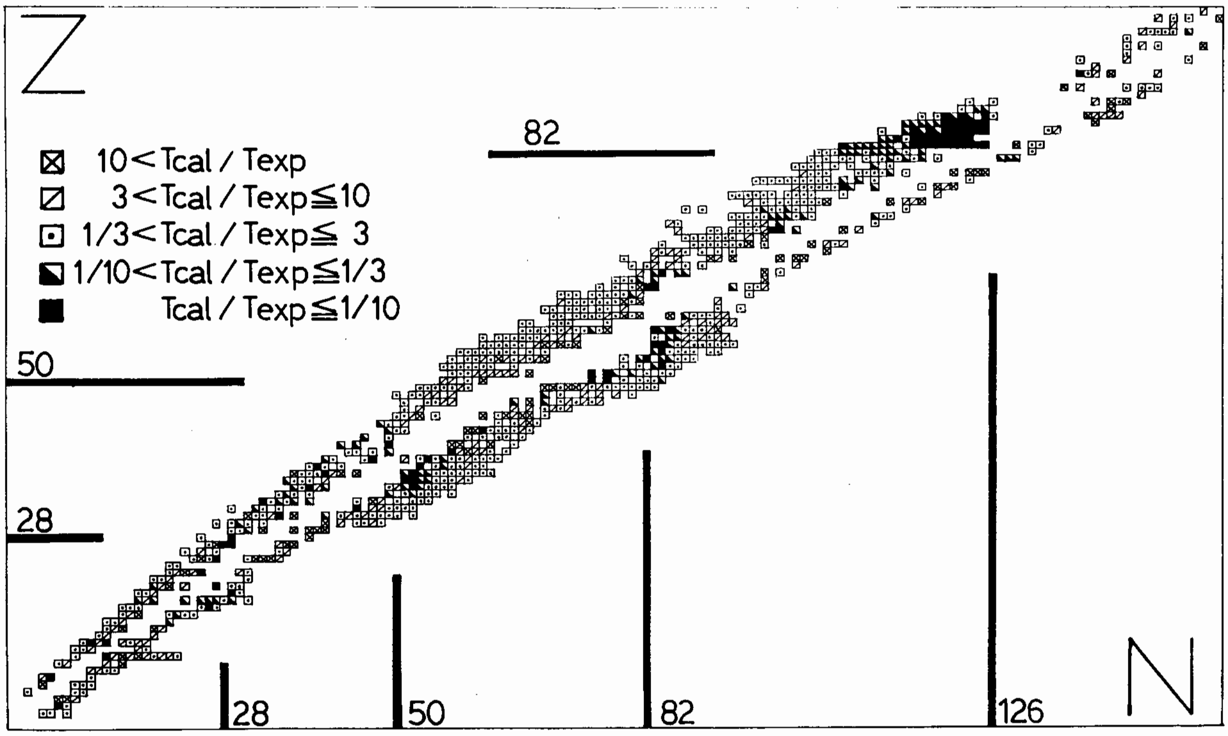

Fig. 8 Comparison of the β-decay half-lives calculated in the improved version of the gross theory with experimental half-lives. Errors in Fig. 5 of Ref. 8 which were brought in by careless drawing have been corrected.

closely related to the "bottom raising" explained in the preceding subsection. Actually, the "bottom raising" is not the only way of modification of the strength function in the low-excitation part as illustrated in Fig.9. A more reasonable way may be to use (c) or (d) of Fig.9 depending on cases. The modification (d) will be particularly important when the spin-parity of the (odd-odd) parent is 1^+. Yoshida

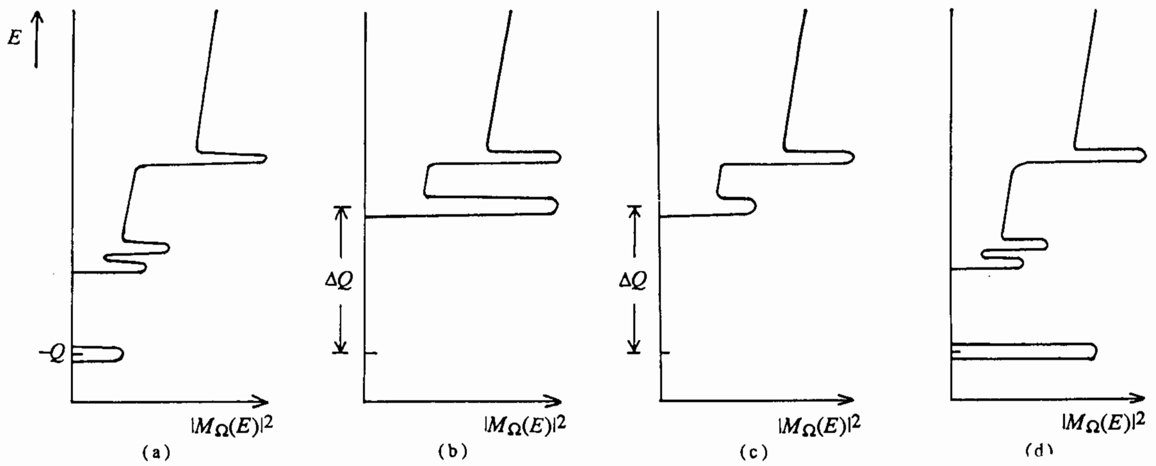

Fig. 9 Schematic illustration of various ways of modification of the β strength function in the low-excitation part. Decay of an odd-odd nucleus is used as an example. (a): Unmodified. (b): Fully "bottom-raised". (c): Partly "bottom-raised" and partly cut down. (d): Ground-state strength increased.

and Nakasima only used (b) of Fig.9, but if we use (c) or (d), then $\overline{E}_\beta$ increases and $\overline{E}_\gamma$ decreases. Therefore, it is likely that we shall get a still more reasonable explanation of $\overline{E}_\beta$ and $\overline{E}_\gamma$ in the improved version of the gross theory.

IV. Beyond the Statistical Theory

The gross theory both in the TYK version and the improved version is a statistical theory in the sense that it gives a β strength function which is expected when we ignore all the peculiarities of individual nuclides other than the Q-value. Microscopic theories take into account such peculiarities, but is there any method for doing so keeping close contact with the gross theory? In the following we discuss such methods.

4.1 Incorporation of shell effects in the gross theory

It is, in principle, possible to incorporate shell effects in the basic equation of the gross theory, Eq.(1). Both $D^{(Q)}_{\Omega}(E,\varepsilon)$ and $dn_1/d\varepsilon$ can include shell effects as discussed by Kondoh and Yamada.[14] However, the process is rather laborious and suffers from a considerable arbitrariness.

4.2 Systematics

Equation (1) suggests a variety of systematics or relations among quantities of neighboring nuclides. Some of them were discussed by Kondoh and Yamada,[14] and later by Tachibana and Yamada.[16~18] According to Tachibana and Yamada, Eq.(1) suggests that, after appropriate shifts of energies, the β strength function of a nucleus (Z_2,N_2) is approximately equal to the sum of the β strength functions of nuclei (Z_1,N_2) and (Z_2,N_1) minus the β strength function of a nucleus (Z_1,N_1) except for the part of the highest transition energy. This relation can be used to estimate an unknown strength function from three measured strength functions.

For neighboring nuclides extremely far from the β stability line, the difference between their β strength functions is likely to be relatively small. Then, the above-mentioned relation among four strength functions suggests that a similar relation may be valid for four half-lives as

$$T^{\beta}_{1/2}(Z_1,N_1)+T^{\beta}_{1/2}(Z_2,N_2)\approx T^{\beta}_{1/2}(Z_1,N_2)+T^{\beta}_{1/2}(Z_2,N_1), \quad (7)$$

or

$$T^{\beta}_{1/2}(Z_1,N_1)\cdot T^{\beta}_{1/2}(Z_2,N_2)\approx T^{\beta}_{1/2}(Z_1,N_2)\cdot T^{\beta}_{1/2}(Z_2,N_1)\,. \quad (8)$$

A relation which is similar to the above but is hopefully valid for nuclides nearer to the β stability line is

$$\frac{T^{\beta}_{1/2}(Z_1,N_1)}{T^{\beta}_{1/2\mathrm{cal}}(Z_1,N_1)}\cdot\frac{T^{\beta}_{1/2}(Z_2,N_2)}{T^{\beta}_{1/2\mathrm{cal}}(Z_2,N_2)}\approx\frac{T^{\beta}_{1/2}(Z_1,N_2)}{T^{\beta}_{1/2\mathrm{cal}}(Z_1,N_2)}\cdot\frac{T^{\beta}_{1/2}(Z_2,N_1)}{T^{\beta}_{1/2\mathrm{cal}}(Z_2,N_1)}\,, \quad (9)$$

where $T^{\beta}_{1/2\mathrm{cal}}$ means the half-life calculated by the gross theory. Equation (9) can also be written as

$$\Delta(Z_2,N_2)\approx\Delta(Z_1,N_2)+\Delta(Z_2,N_1)-\Delta(Z_1,N_1)\,, \quad (10)$$

with

$$\Delta(Z,N)=\log[T^{\beta}_{1/2}(Z,N)/T^{\beta}_{1/2\mathrm{cal}}(Z,N)]\,, \quad (11)$$

Approximate relations like Eq.(10) are also expected to be valid in some other cases. Generalizing the meaning of $\Delta(Z,N)$, we take $\Delta(Z,N)$ as a quantity to represent the difference between the true and calculated values. Then, it is likely that $\Delta(Z,N)$ is often approximated by a sum of proton shell and neutron shell terms, at least in a local nuclidic region, as

$$\Delta(Z,N)=X_Z+x_ZN+Y_N+y_NZ\,, \quad (12)$$

where X_Z+x_ZN represents the proton shell effect, and Y_N+y_NZ the neutron shell effect. Since Z and N are usually much larger than unity, x_Z and y_N are generally much smaller than X_Z and Y_N. From Eq.(12), we get

$$\Delta(Z_2,N_2)=\Delta(Z_1,N_2)+\Delta(Z_2,N_1)-\Delta(Z_1,N_1)$$
$$+(x_{Z1}-x_{Z2})(N_1-N_2)+(y_{N1}-y_{N2})(Z_1-Z_2)\,. \quad (13)$$

If we take $|N_1-N_2|=1$ to 2 and $|Z_1-Z_2|=1$ to 2, the last two terms of Eq.(13) are much smaller than $\Delta(Z_i,N_j)$, and the approximate equation (10) follows. Equation (10) can be used to estimate the quantity of the nuclide (Z_2,N_2) from $\Delta(Z_1,N_1)$, $\Delta(Z_1,N_2)$, $\Delta(Z_2,N_1)$ and the calculated value of the nuclide (Z_2,N_2).

A more general expression for $\Delta(Z,N)$ may be

$$\Delta(Z,N)=X_Z+x_Z N+Y_N+y_N Z,+\Delta_{\rm rnd}(Z,N), \quad (14)$$

where $\Delta_{\rm rnd}(Z,N)$ behaves in an apparently random way on the N-Z plane. In this case if we use Eq.(10) to predict $\Delta(Z_2,N_2)$, we shall overestimate $|\Delta(Z_2,N_2)|$ on the average. Then, a more refined formula would be

$$\Delta(Z_2,N_2)=\frac{1}{ab}\sinh^{-1}\{a[\sinh^{-1}(b\Delta(Z_1,N_2))$$
$$+\sinh^{-1}(b\Delta(Z_2,N_1))-\sinh^{-1}(b\Delta(Z_1,N_1))]\}. \quad (15)$$

In the limit of $a,b\rightarrow 0$, Eq.(15) reduces to Eq.(10).

We have made a preliminary test of the usefulness of the above method of estimation with known even-even nuclei. We have estimated $T^{\beta-}_{1/2}(Z_2,N_2)$ from the following seven quantities: $T^{\beta-}_{1/2\rm exp}$ of (Z_2+2,N_2), (Z_2,N_2-2), (Z_2+2,N_2-2) nuclei and $T^{\beta-}_{1/2\rm cal}$ of all these four nuclei. We have used Eq.(15) with a=0.9375 and $b\approx1.6$ and with "log" representing natural logarithm. We denote this estimated half-life by $T^{\beta-}_{1/2\rm est}(Z_2,N_2)$. For β^+-decay and electron capture, $T^{\beta+}_{1/2}(Z_2,N_2)$ have been estimated similarly with use of the data of (Z_2-2,N_2), (Z_2,N_2+2), (Z_2-2,N_2+2) nuclei. Then, the root-mean-square of $\log_{10}[T^{\beta}_{1/2\rm est}(Z_2,N_2)/T^{\beta}_{1/2\rm exp}(Z_2,N_2)]$, where $T^{\beta}_{1/2\rm exp}$ means experimental half-life, has been obtained to be 0.295, while that of $\log_{10}[T^{\beta}_{1/2\rm cal}(Z_2,N_2)/T^{\beta}_{1/2\rm exp}(Z_2,N_2)]$ is 0.43. Thus, a considerable improvement has been achieved in this case. For odd-mass and odd-odd nuclei, some additional caution is necessary in relation to their spin-parities. Anyway, Eqs.(10) and (15) are very simple and worth trying to use when one wants to estimate the β-decay half-life of an unknown nucleus.

V. Conclusion

We have modified the gross theory of β-decay, and succeeded in improving the agreement between theory and experiment. Further improvements may be made in the future. However, if the gross theory is only to give the average properties of β-decay, the improved version explained in this article is likely to be close to the best form in the sense that further changes in the calculated average quantities will probably remain small. Then, the next efforts will mainly be directed to explanation and prediction of the properties of individual nuclides. In addition to the microscopic calculations which are generally regarded as appropriate for such a purpose, there seems to be another approach which is based on approximate relations among true (experimental) values and calculated values of several neighboring nuclides, as exemplified in Subsection 4.2.

More details of the results of numerical calculations will be sent upon request.

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